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$$=\tfrac{1}{4}\int_0^{\frac{1}{2}\pi} r^4 \sin\theta \sin(\theta+\beta)\,d\theta$$

$$=4a^4b^8\int_0^{\frac{1}{2}\pi} \frac{(\cos^4\theta \sin^2\theta \cos\beta+\cos^5\theta \sin\theta \sin\beta)\,d\theta}{(a^2\sin^2\theta+b^2\cos^2\theta)^4},$$

$$=\tfrac{1}{2\pi}ab^2\,(16a\sin\beta=3\pi\, b\cos\beta).$$

$$\bar{x}=M/N=\tfrac{1}{4}a.\frac{16b\cos\beta+15a\pi\sin\beta}{4b\cos\beta+3a\pi\sin\beta},\quad \bar{y}=P/N=\tfrac{1}{4}b.\frac{16a\sin\beta+3b\pi\cos\beta}{4b\cos\beta+3a\pi\sin\beta}.$$

If $\beta=0$, $\bar{x}=a$, $\bar{y}=3\pi b/16$.

Also solved by **S. Lefsehetz.**

230. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A particle is projected from a distance $a=2r$ from the earth's center towards the earth with a velocity from infinity. If the earth were an airless homogeneous sphere, radius equal to the present mean radius and gravity as at present, with what velocity and in what time would it reach the center through an opening from surface to center?

Solution by the PROPOSER.

$\int_{v_1}^{v} vdv=\int_x^a fdx$, but $f=\frac{gr^2}{x^2}$ above the earth where r is earth's radius.

$$\therefore \int_{v_1}^{v} vdv=gr^2=\int_x^a \frac{dx}{x^2}.\quad \therefore \tfrac{1}{2}(v^2-v_1^2)=\frac{gr^2\,(a-x)}{ax}.$$

$$\therefore v^2=\frac{2gr^2\,(a-x)}{ax}+v_1^2=\left(\frac{dx}{dt}\right)^2,$$ where v_1 is the initial velocity.

$$\therefore t=\sqrt{a}\int_r^a \frac{\sqrt{(x)}\,dx}{\sqrt{[2agr^2+x(av_1^2-2gr^2)}}.$$

If $v_1=r\sqrt{\frac{2g}{a}}$, $t=\frac{2(a^{\frac{3}{2}}-r^{\frac{3}{2}})}{3r\sqrt{(2g)}}$, $v=\sqrt{(2gr)}$ at surface.

$$\text{If } v_1<r\sqrt{\frac{2g}{a}},\ t=2gr^2\sqrt{\left(\frac{a}{2gr^2-av_1^2}\right)^3}\left(\cos^{-1}\sqrt{\frac{2gr^2-av_1^2}{2agr}}\right.$$

$$-\cos^{-1}\sqrt{\frac{2gr^2-av_1^2}{2gr^2}}+\frac{\sqrt{[(2gr^2-av_1^2)(2agr+av_1^2-2gr^2)]}}{2agr}$$

$$-\frac{v_1\sqrt{[a(2gr^2-av_1^2)]}}{2gr^2}\Bigg).$$

If $v_1 < r\sqrt{\frac{2g}{a}}$,

$$t=2gr^2\sqrt{\left(\frac{a}{av_1^2-2gr^2}\right)^3}\Bigg\{\frac{v_1\sqrt{[a(av_1^2-2gr^2)]}}{2gr^2}$$

$$-\frac{\sqrt{[(av_1^2-2gr^2)(2agr+av_1^2-2gr^2)]}}{2agr}$$

$$+\log\left[\sqrt{\frac{a}{r}}\left(\frac{\sqrt{(av_1^2-2gr^2)}+v_1\sqrt{a}}{\sqrt{(av_1^2-2gr^2)}+\sqrt{(2agr+av_1^2-2gr^2)}}\right)\right]\Bigg\}.$$

Since $v_1=\sqrt{(2gr)}$ =velocity from infinity, and $a=2r$, $v_1>r\sqrt{(2g/a)}$. Hence time of falling to surface is

$$t=2\sqrt{\frac{r}{g}}\left(\frac{2\sqrt{2}-\sqrt{3}}{2}+\log\frac{2+\sqrt{2}}{1+\sqrt{3}}\right).$$

Velocity at the surface becomes

$$v_2^2=2gr^2\left(\frac{2r-r}{2r^2}\right)+2gr=3gr.\qquad \therefore v_2=\sqrt{(3gr)}.$$

The formula for below the earth's surface is

$$\int_{v_2}^{v_3} v_3\,dv=\int f\,dx=\frac{g}{r}\int_0^x (r-x)\,dx.\qquad \therefore v_3^2-v_2^2=\frac{g}{r}(2rx-x^2).$$

$$\therefore v_3^2=(g/r)(2rx-x^2)+v_2^2=(dx/dt)^2.$$

$$\therefore t_1=\int_0^r\frac{dx}{\sqrt{[v_2^2+(g/r)(2rx-x^2)]}}=\sqrt{\frac{r}{g}}\sin^{-1}\left(\frac{r\sqrt{g}}{\sqrt{(v_2^2r+gr^2)}}\right).$$

Hence time of falling from surface to center is $t_1=\frac{\pi}{6}\sqrt{\frac{r}{g}}$.

Since $v_2=\sqrt{(3gr)}$, $v_3=2\sqrt{(gr)}$, the velocity at the center when $x=r$.

Taking for the mean semi-axes of the earth the values 20926202 feet and 20854895 feet, the radius of a sphere of equal volume is 20902410 feet, and the acceleration g at the surface of this sphere is 32.10614 feet.

$\therefore \sqrt{(r/g)}=806.871.$

$T=t+t_1=\sqrt{\frac{r}{g}}\left(\frac{1}{6}\pi+2\sqrt{2}-\sqrt{3}+2\log\frac{2+\sqrt{2}}{1+\sqrt{3}}\right)=1666.673$ seconds $=27$ minutes, 46.673 seconds.

$v_3=44869.668$ feet per second$=8.498$ miles$=8\frac{1}{2}$ miles per second, nearly.

When $v_1=r\sqrt{(2g/a)}$, $v_2=\sqrt{(2gr)}$, and the velocity of arriving at the surface is independent of a, the distance from the center.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

163. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

Prove that the equation $y^n=mx+1$ always has at least one positive integer solution (different from $y=1$, $x=0$), whatever integer values m and n may have.

Solution by S. LEFSEHETZ, Pittsburg, Pa.

The following solution is evident:

$$y=(m+1)^n,\quad x=\frac{(m+1)^n-1}{m}.$$

To find all solutions, we remark that it is enough to find all values of y such that $y^n\equiv1 \pmod{m}$. Let f be a divisor of $\phi(m)$, a number to which f appertains. Then $a^f\equiv1 \pmod{m}$. If also $a^n\equiv1 \pmod{m}$, we must have $n\equiv0 \pmod{f}$. Hence, f is a divisor of $dv[n,\ \phi(m)]$. Therefore we take the $\phi(m)$ numbers smaller than m and prime to it, we form the exponents to which they appertain and keep them if their exponents divide $dv[n,\ \phi(m)]$. If x be such a value, $y=x+km$ is a solution, the corresponding value of x being $\frac{y^n-1}{m}$, which is integral since $y^n\equiv1 \pmod{m}$.

AVERAGE AND PROBABILITY.

200. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

A line $AB=l$ is extended to P making $BP=p$. If a point D is taken at random in BP, what is the mean value of $AD.DP$?

Solution by J. EDWARD SANDERS, Weather Bureau, Chicago, Ill.

Let $x=BD$. Then $AD=l+x$, $DP=p-x$, and $AD.DP=(l+x)(p-x)$.

$$\therefore M=\int_0^p (l+x)(p-x)dx \Big/ \int_0^p dx=\frac{1}{p}\int_0^p[lp+(p-l)x-x^2]dx$$

$$=\tfrac{1}{6}p(3l+p).$$

Also solved by G. B. M. Zerr.